Some Features of Dedicated T1-Filters

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Abstract. *This report continues developments* of T_1 *-filters aimed for the contrast enhancing in NMR imaging. The characteristic property of the proposed technique is that only 90 and 180 degree pulses are used. The mathematical expressions were presented for transfer functions, time and frequency responses of these filters.*

Keywords: T_1 *Filters, NMR, Transfer Functions, Time and Frequency Responses*

1. Introduction

Recently T_1-filters were proposed [1, 2, 3] for the contrast enhancing in NMR-imaging. The structure of the T_1-filters can be described as

$$90^\circ - \tau_1 - 180^\circ - \tau_2 - 180^\circ - \bullet\bullet\bullet 180^\circ - \tau_N - 90^\circ \tag{1}$$

The pulse sequences used for the T_1-filters were named DIFN (differentiation by N exciting radiofrequency pulses) keeping in mind the number of the time intervals between the all pulses and the number of pulses without the last reading 90° pulse. The number of 180° pulses equals to $n = N - 1$. Here n and N are integer numbers.

The aim of the paper is to compare some features of the T_1-filters with the features of the low-pass Butterworth filters [4].

2. Subject and Methods

One of the features of the polynomials is that the coefficients located symmetrically relative to "the center of gravity" of the polynomials are pairwise equal. These coefficients are calculated by multiplying the factors of the polynomials. The factors are defined by the poles of polynomials. The n poles of the Butterworth filters occur on a circle of radius ω_c (the cut-off frequency) at equally spaced points, and are symmetric around the imaginary axis. The transfer function, $H(s)$, contains only the poles in the negative real half-plane of s.

The frequency response of the Butterworth low-pass filter is maximally flat in the passband and rolls off towards zero in the stopband. The slope of the frequency response $G(\omega) = |H(j\omega)|$ at frequencies higher than the cutoff frequency equals to - $n20$ dB/decade.

Various methods are used to make T_1-selective pulse sequences in NMR and NMR-imaging. T_1-filters were proposed for the contrast enhancing. They can be also applied for separation of multicomponent relaxation curves in liquids, heterogeneous and multiphase objects.

The response of the longitudinal magnetization to the pulse sequence (1) is calculated in the following manner. The behavior of the longitudinal magnetization being initially in an equilibrium after excitation by a 90°pulse can be described by

$$M(t) = M_0 - (M_0 - M(0_+))\exp(-\frac{t}{T_1}), \quad (3)$$

where $M(0_-)$ and $M(0_+)$ are the magnetization values prior to and after the exciting pulse.

$$M(0_+) = M(0_-)cos\beta. \quad (4)$$

If $\beta = 90^\circ$, $cos\beta = 0$, and for the saturation-recovery (SR) pulse sequence

$$M(0_+) = 0, M_z(t) = M_0\left[1 - \exp\left(-\frac{\tau}{T_1}\right)\right] = M_0\,[1 - \mathrm{E}(\tau)]. \quad (5)$$

Here the notation $\mathrm{E}(\tau) = \exp\left(-\frac{\tau}{T_1}\right)$ was introduced.

Then, if $\beta = 180^\circ$, $cos\beta = -1$, and for the inversion-recovery (IR) pulse sequence

$$M_z(t) = M_0\left[1 - 2\exp\left(-\frac{\tau}{T_1}\right)\right] = M_0\,[1 - 2\mathrm{E}(\tau)]. \quad (6)$$

Calculations can be simplified by introducing the designation $E_i = E(\tau_i)$ and letting $M_0 = 1$.

For $T_1, \tau \gg T_2$ only M_z can be considered.

After the pulse sequence (1) with $i = N$ time intervals, the magnetization equals to

$$M_z(\tau_i) = 1 - 2E_i + 2E_iE_{i-1} - 2E_iE_{i-1}E_{i-2} + \cdots + (-1)^iE_iE_{i-1}E_{i-2}\ldots E_1. \quad (7)$$

The pulse sequences were tested and the time intervals were modified in analog to the electronic and digital filters. After optimization, they became

$$90^\circ - \tau_0 - 180^\circ - \tau_0 - 90^\circ\,; \quad \text{DIF2}$$

$$90^\circ - \tau_0 - 180^\circ - 2\tau_0 - 180^\circ - \tau_0 - 90^\circ\,; \quad \text{DIF3}$$

$$90^\circ - \tau_0 - 180^\circ - \tau_0(\sqrt{2} + 1) - 180^\circ - \tau_0(\sqrt{2} + 1) - 180^\circ - \tau_0 - 90^\circ\,; \quad \text{DIF4}$$

$$90^\circ - \tau_0 - 180^\circ - \tau_0(\sqrt{3} + 1) - 180^\circ - \tau_0(\sqrt{3} + 2) - 180^\circ - \tau_0(\sqrt{3} + 2) -$$

$$-180^\circ - \tau_0(\sqrt{3} + 1) - 180^\circ - \tau_0 - 90^\circ\,. \quad \text{DIF6}$$

Here τ_0 is the variable time interval between the first two pulses relative to which the other intervals are calculated. The behavior of the magnetization after these pulse sequences represented in fig. 1. It can be seen from Eq. (7) and Fig.1 that M_z vs t curves are similar to aperiodic unit-step responses which in turn are the linear combinations of the N aperiodic unit-step responses. Therefore the aperiodic unit-step response of the T_1-filters can be presented as

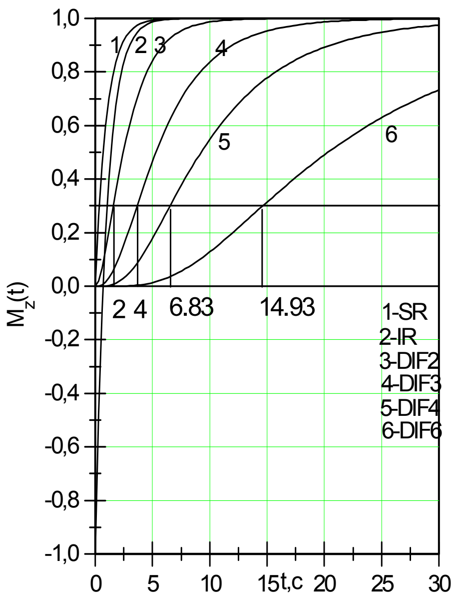

Fig.1. Recovery of normalized longitudinal magnetization after the act of pulse sequences: the horizontal line cuts off 0.3 of the full normalized magnetization: figures near the middle coordinate line are equal to the full duration of an NMR signal before the reading pulse

$$M_z(\sum k_i\tau_0) = 1 - \sum_{i=1}^{N} a_i \exp[-\frac{k_i\tau_0}{T_1}] , \tag{8}$$

where k_i are the coefficients in front of $\frac{\tau_0}{T_1}$ after multiplication of E_i and summation of the time intervals in the exponents, a_i are the coefficients in front of E_i and products of $E_iE_{i-1}E_{i-2} \dots E_1$ in the Eq. (7).

The corresponding to (8) impulse response is

$$\frac{d}{d\tau_0}[M_z(\sum k_n\tau_0)] = \sum_{n=1}^{N} \frac{k_n}{T_1} a_n \exp[-\frac{k_n\tau_0}{T_1}] = \sum_{n=1}^{N} \frac{a_n}{T'_{1n}} \exp[-\frac{\tau_0}{T'_{1n}}],$$

where the designation $T'_{1n} = \frac{T_1}{k_n}$ is introduced.

According the control theory, the transfer functions of the T_1-filters can be presented as

$$W(p) = \sum_{n=1}^{N} \frac{a_n}{T'_{1n}} \frac{1}{(pT'_{1n}+1)}. \tag{9}$$

The correspondent frequency responses can be described as

$$G(\omega) = \sum_{n=1}^{N} \frac{a_n}{T'_{1n}} \frac{1}{\sqrt{(T'_{1n}\omega)^2+1}} . \tag{10}$$

3. Results and Discussion

Hence, the transfer functions of the T_1-filters and the correspondent frequency responses are also the algebraic sums (with alternative signs) of those for the first-order aperiodic units.

The time intervals between the pulses in the T_1-filters located symmetrically relative to "the center of gravity" of the pulse sequences are also are equal, as they are in the Butterworth filters.

In the analogy to the Butterworth filters, the time intervals in the T_1-filters are related with the equally spaced positions of the n points on a circle of unit radius (Fig. 2)

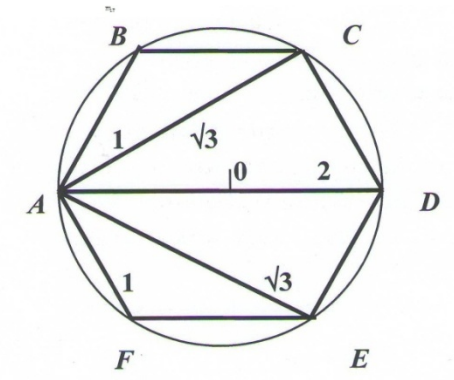

Fig.2. Positions and lengths of the segments of the 6-sided inscribed polygon which constitute (add up to) the time intervals in the *DIF*6 pulse sequence

For the example, let us consider the DIF6 pulse sequence. The time intervals in *DIF*6 pulse sequence equal to the lengths of the segments and consequent pairwise sums of the lengths of the segments of the 6-sided inscribed polygon which originate from one point, A: $|AB|$, $|AB|+|AC|$, $|AC|+|AD|$, $|AD|+|AE|$, $|AE|+|AF|$, $|AF|$, in the units of $|AB|$.

4. Summary and Conclusions

It was shown in this report that T_1-filters developed for contrast enhancing in NMR-imaging and differentiating of multiexponential relaxation curves have some relation to the low pass Butterworth filters. The mathematical expressions were presented for transfer functions, time and frequency responses of these filters.

The results reported can be used as a basis to develop more complicated pulse sequences for the synthesis of pass-band, stop-band, high-pass band and other filters in the longitudinal relaxation.

5. References

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