# A Method for the Determination of Thermal Time Constant of Pyroelectric Sensor from Voltage Response to Step Optical Input Signal

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**Abstract**. A method for measuring of the thermal time constant of pyroelectric sensor using the voltage response to a step optical input signal is proposed. An important advantage of this method over the traditional one is, that to get a correct result of the thermal time constant measurement it is no longer required that the thermal and electric time constants should be significantly different.

Keywords: Pyroelectric Sensor, Responsivity, Thermal Time Constant of Pyroelectric Sensor

## 1. Introduction

The key parameters determining the performance of pyroelectric sensors are the thermal and electric time constants. The electric time constant can be relatively easily predicted already at the stage of design and later experimentally verified, but analytical determination of the thermal time constant is usually much more difficult. The problem is that the thermal energy absorbed by the sensor is passed to the environment simultaneously via three heat transporting processes: conduction, convection and radiation, which is rather difficult of exact modelling. Consequently, thermal time constant is usually determined in experiment.

In general, the problem of experiments performed to establish the thermal time constant of a pyroelectric sensor has been rarely presented in research reports and has been described in a brief manner. Usually, the thermal time constant,  $\tau_{th}$ , is found on the basis of the Bode plot of voltage sensitivity of the sensor  $R_V$  as a function of frequency  $\omega$  of the sinusoidal signal of radiation exciting the pyroelectric sensor. The thermal time constant is calculated from the relation  $\tau_{th} = 1/\omega_{th}$  with  $\omega_{th}$  obtained from the Bode plot as corresponding to 3 dB limit of the frequency band, [1, 2]. This method of measurement is illustrated in Fig. 1.



Fig. 1. The principle of thermal time constant determination with the use of Bode plot.

The above method has significant limitations as the relation  $\tau_{th} = 1/\omega_{th}$  holds only when the thermal time constant  $\tau_{th}$  and electric time constant  $\tau_e$  take significantly different values. Moreover, this method is rather technically difficult as the majority of the commercially

available electromechanical modulators of radiation are designed to produce rectangular or trapezium shape signals of radiation.

The paper presents a new method proposed for determination of the thermal time constant of a pyroelectric sensor based on the use of the peak voltage response of the sensor to the radiation signal which is of the unit step function. The proposed equation derived for calculation of the thermal time constant permits determination of this constant without the condition that the thermal and electric time constant must be significantly different. Another advantage of this method is the use in experiment of an easily generated step signal (e.g. rectangular) of optical radiation to excite the sensor.

#### 2. Subject and Methods

In general, analytical considerations and implied the final form of the equation describing the voltage response V(t) of the sensor to excitation by step optical signal of the amplitude of radiation power  $\Phi_m$  are known and have been described in many papers, e.g. [3]. The equation of voltage response V(t) can be derived in several different ways but the mathematical form of the final equation is similar in the most of articles. Therefore, in this paper only the main mathematical expressions and the most important explanation are given. Usually the starting point of such analyses is the equivalent circuit of the pyroelectric sensor[3, 4] and the corresponding mathematical description (Fig. 2).



Fig. 2. a) Equivalent circuit of a pyroelectric sensor cooperating with voltage amplifier. b) Transfer function model of sensor[3,4]. Symbols in Fig.1:  $C_{th}$  – thermal capacity of pyroelectric sensor,  $G_{th}$  – thermal conductance of pyroelectric sensor,  $\eta$  – absorption coefficient of radiation, p – pyroelectric coefficient,  $\Delta T(t)$  – temperature changes of pyroelectric material, C – equivalent capacitance for parallel connected pyroelectric capacitance  $C_d$  and input amplifier capacitance  $C_L$ :  $C = C_d + C_L$ , R - equivalent resistance for parallel connected leakage resistance  $R_d$  of pyroelectric sensor and input amplifier resistance:  $R = R_d$   $R_L/(R_d + R_L)$ ,  $\tau_{th}$  – thermal time constant  $\tau_{th} = C_{th} / G_{th}$ ,  $\tau_e$  - electric thermal time constant  $\tau_e = CR$ ,  $R_v(s)$ , A - active surface of the sensor,  $R_v(s)$  – voltage responsivity of sensor in the Laplace domain.

The equivalent circuit of pyroelectric sensor presented in Fig.2 a permits modelling of the process of transformation of the radiation power signal  $\Phi(t)$  absorbed by the pyroelectric into a voltage signal V(t) passed to the input of the amplifier. This process is composed of the three stages of conversion: thermal, thermoelectric and electric. Each stage is described by appropriate differential equation and the corresponding equation in the Laplace domain, this problem is described in detail in [3,4]. After necessary mathematical transformations of the Laplace domain equations (Fig. 2) a mathematical model of the pyroelectric sensor can be obtained in the form of a Laplace transfer function  $R_V(s)=V(s)/\Phi=p\eta\tau_{th}\tau_e s/CC_{th}(s\tau_{th}+1)(s\tau_e+1)$  (Fig 2b), which describes the relation between the output signal V(s) and the input signal  $\Phi(s)$  for the circuit presented in Fig. 2a. If the input signal is described by the relation of the unit

step type,  $\Phi(t) = \Phi_m \mathbf{1}(t)$ , then as a result of the inverse Laplace transformation we get the equation describing the voltage response V(t) of the pyroelectric sensor as

$$V(t) = \pounds^{-1}[\Phi(s)R_{\nu}(s)] = \frac{pA\eta\Phi_{m}\tau_{e}\tau_{th}}{CC_{th}(\tau_{e} - \tau_{th})} (e^{-t/\tau_{e}} - e^{-t/\tau_{th}}), \quad \tau_{th} \neq \tau_{e}$$
(1)

Similar results for the voltage response of a pyroelectric sensor to a step signal of optical radiation have been presented e.g. in [5].

Fig. 3 presents an exemplary plot of the pyroelectric sensor voltage response V(t) to a step signal of optical radiation. Parameters of the sensor were specified in Fig. 3.



Fig. 3. Voltage response V(t) of pyroelectric sensor to step signal of optical radiation.

time  $t_{\text{max}} = [\tau_e \tau_{th} / (\tau_{th} - \tau_e)] ln(\tau_{th} / \tau_e)$  [5]. After transformation of this relation we get the following equation:

$$\ln\frac{\tau_{th}}{\tau_e} + \frac{t_{\max}}{\tau_{th}} - \frac{t_{\max}}{\tau_e} = 0.$$
<sup>(2)</sup>

Equation (2) is of key significance for determination of the thermal time constant of a pyroelectric sensor  $\tau_{th}$ . If the value of  $t_{max}$  is known from the experimentally found voltage response to a single step signal of optical radiation, V(t), and if the electric time constant  $\tau_e$  is known from the parameters of the sensor and amplifier working with it, then by solving equation (2) we find  $\tau_{th}$ . Equation (2) can be easily solved numerically using appropriate computer applications.

#### Results

Figs. 4a and 4b show the plots of two time dependencies of the sensor voltage response to the step signal of optical radiation. The plot in Fig. 4a was obtained experimentally on the basis of measurements for a specific sensor of known material and size parameters. The recorded signal of voltage response (Fig. 4a) with use of the digital oscilloscope permits determination of  $t_{\text{max}}$ . With the known  $t_{\text{max}} = 9$  ms and  $\tau_e = 4.7$  ms, the value of the thermal time constant can

be calculated from eq. (2) as  $\tau_{th} \approx 20$  ms. The second normalised plot, shown in Fig. 4b, is obtained on the basis of the theoretical relation (2) for the calculated thermal time constant  $\tau_{th} = 20$  ms and all other parameters whose values were determined from the known construction features of the sensor and catalogue data for the pyroelectric material applied. The plots presented in Figs. 4a and 4b show the acceptable similarity in shape, especially in the time range of the plot rising edge to the maximum value.



Fig. 4. Voltage response of the pyroelectric sensor to a single step signal of optical radiation a) obtained experimentally, b) theoretical.

#### 3. Conclusions

The proposed method for the measurement of the thermal time constant of the pyroelectric sensor on the basis of its voltage response to single step optical radiation signal can be a competitive alternative to the usually applied methods employing the frequency dependencies obtained for a sinusoidal signal. The method proposed is easy to perform with the help of typical modulators. The subsequent operations include recording of the voltage response of the sensor, determination of the time at which the response reaches the maximum value and necessary calculations.

### References

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