Analysis of the Activation Propagation Velocity in the Slab Model of the Cardiac Tissue

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Abstract. *In the study, the activation propagation velocity in cardiac tissue was simulated in COMSOL Multiphysics environment using the modified FitzHugh-Nagumo model of the electrical excitation. The influence of different model parameters and stimulation conditions on the activation propagation velocity was evaluated. The homogeneous slab model was used as the model of the atrial wall. Results of simulations could help to explain the differences in activation propagation velocities in measured data.*

Keywords: Myocardium, Monodomain Model, Activation Propagation Velocity

1. Introduction

Electrophysiological activity of human cardiac cells may be modelled using local or space membrane models. The space models suitable for simulation of electrical activation propagation are based on reaction - diffusion equations in monodomain or bidomain models [1]. Electrical activation in the monodomain model of the cardiac tissue is described by the partial differential equation:

$$\frac{\partial V_m}{\partial t} = \frac{1}{\beta C_m}\nabla\cdot\left(\sigma\nabla V_m\right) - \frac{1}{C_m}\left(I_{ion} + I_s\right) \tag{1}$$

where V_m is the membrane potential
β is the membrane surface-to-volume ratio
C_m is the membrane capacitance per unit area
σ is the tissue conductivity
I_ion is the ionic transmembrane current density per unit area and
I_s is the stimulation current density per unit area.

Using substitutions

$$D = \frac{\sigma}{\beta C_m}, \qquad i_{ion} = \frac{I_{ion}}{C_m} \qquad \text{and} \qquad i_s = \frac{I_s}{C_m} \tag{2), (3), (4)}$$

the equation (1) describing the time change of the membrane potential V_m can be rewritten in the form:

$$\frac{\partial V_m}{\partial t} = \nabla\cdot\left(D\nabla V_m\right) - i_{ion} + i_s \tag{5}$$

The local membrane models could be obtained from (5) by omitting the space derivatives:

$$\frac{\partial V_m}{\partial t} = -i_{ion} + i_s \tag{6}$$

The local membrane properties of atrial cells may be modelled using various less or more complex models, e.g. the Courtemanche-Ramirez-Nattel (CRN) model [2] or the modified FitzHugh-Nagumo (FHN) model [3] - [6]. In the CRN model, the ionic current comprises different membrane currents, e. g. the fast sodium current, calcium, potassium and other membrane currents. Such physiological models could comprise tens of dependent variables and so tens of ordinary differential equations.

If using the less computationally demanding equations of the modified FHN model, the normalized ionic transmembrane current density i_{ion} from equation (6) is:

$$i_{ion} = k\,c_1(V_m - B)\left(-\frac{(V_m - B)}{A} + a\right)\left(-\frac{(V_m - B)}{A} + 1\right) + k\,c_2 R(V_m - B) \tag{7}$$

and

$$\frac{d\,R}{d\,t} = k\,e\left(\frac{(V_m - B)}{A} - R\right) \tag{8}$$

where R is the recovery variable,
a is relating to the excitation threshold,
e is relating to the excitability,
A is the action potential amplitude,
B is the resting membrane potential, and
c_1, c_2 and k are the other membrane-specific parameters.

2. Subject and Methods

In the article, the atrial wall was approximated by SLAB 1 or SLAB 2 model of size 50 x 50 x 2 mm or 5 x 50 x 2 mm (i. e., the thickness of the atrial wall was w = 2 mm). The 1D propagation in SLAB 2 represented a plane wave, while the 2D propagation in SLAB 1 represented a convex (circular) wave front.

If not mentioned otherwise, as the default model parameters were used modified FHN atrial membrane parameters from [5]: a = 0.13, c_1 = 2.6, c_2 = 1, e = 0.0132, k = 1000 s^{-1}, A = 0.120 V, B = -0.085 V, D = 0.0005 m^2/s (relating to σ = 0.5 S/m) and i_s = 40 A/F (lasting 0.002 s). Stimulation current was applied in the centre of the SLAB 1 model in the cylindrical area, with base radius of 2 mm or at one side of the SLAB 2 model, in the area where x <= 2 mm (gray areas in Fig. 1). Initial values of the membrane potential and the recovery variable were -0.085 V and 0, respectively.

Models were numerically solved using the FGMRES iterative solver in COMSOL Multiphysics environment. The zero Neumann boundary condition was used, with exception of the two lateral boundaries in the SLAB 2 with periodic boundary conditions (Fig. 1).

Velocity of activation propagation was evaluated for default FHN model parameter values in both SLAB1 and SLAB 2 models, as well as for 20 % decrease and 20% increase of selected parameters in SLAB 2 model (Table 1).

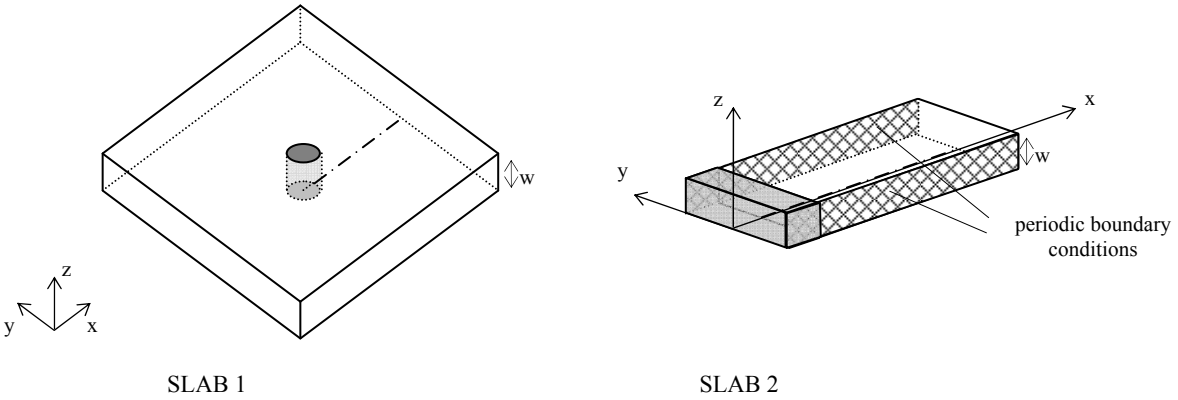


Fig. 1. The atrial wall models: SLAB 1 (left), with coordination system centered in the middle of the stimulated cylinder, and SLAB 2 (right).

3. Results

Examples of the membrane potential V_m distribution in the SLAB 1 and SLAB 2 models are shown in Fig. 2.

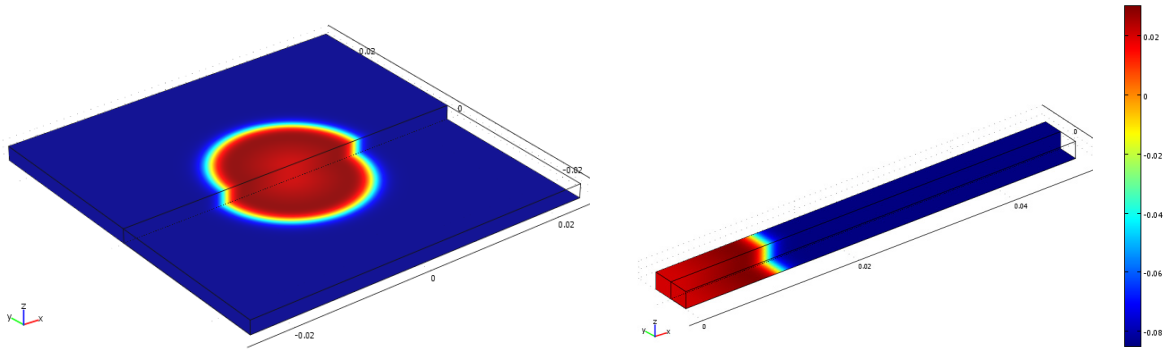
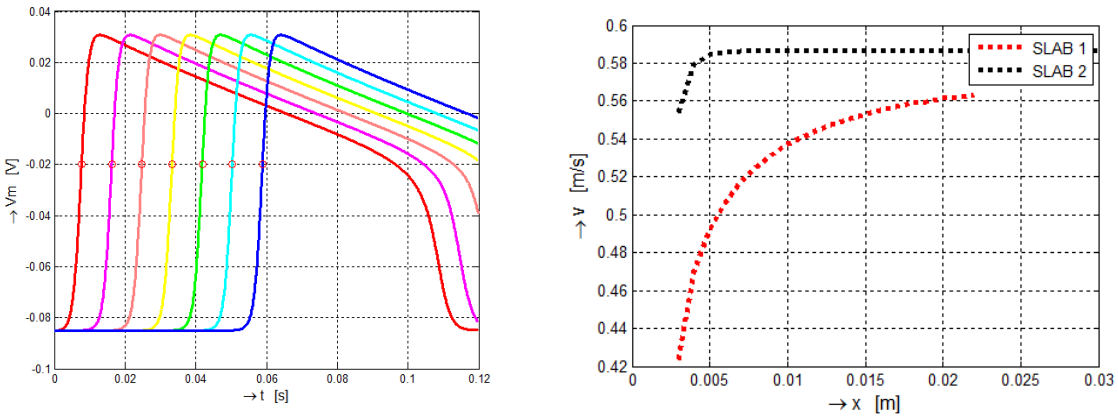


Fig. 2. Distribution of the membrane potential V_m [V] in the SLAB 1 (left), and SLAB 2 (right) in time 0.02 s after stimulation onset (activated areas are in the middle and on the left of the slabs).

Activation propagation velocity was determined from action potentials (APs) obtained in separate points along the positive x axis (shown as dot-and-dashed lines in Fig. 1) using time instants when AP crossed the -20 mV value (marked by circles in Fig. 3A). Activation propagation velocity v in SLAB 1 in the area near to stimulation area was smaller than in SLAB 2 (Fig. 3 B). The velocity of activation propagation in SLAB 1 increased with the distance from the stimulation area. This is in accordance with data measured in [7] and this phenomenon relates to the convex curvature of activation front.

For the default values of FHN model parameters, activation propagation velocity in SLAB 2 reached a constant value of 0.586 m/s within few millimetres away from the stimulation area (Fig. 3 B).

Further, in SLAB 2 the activation propagation velocity v was evaluated also for 20 % decrease and 20% increase of selected FHN model parameter values, results are shown in Table 1.



A B

Fig. 3. Time courses of the membrane potential V_m for default FHN model parameters at points x = 0.005 m, 0.010 m, ..., 0.035 m in the SLAB 2 (A), dependence of activation propagation velocity v on the distance from of the stimulated area in the SLAB 1 and SLAB 2 (B).

Table 1. Sensitivity of the propagation velocity v on the FHN model parameters in the SLAB 2 model.

	20% decrease of selected parameter		20% increase of selected parameter	
Parameter	v [m/s]	Δv [%]	v [m/s]	Δv [%]
c_1	0.519	-11.5	0.646	10.1
k [s^{-1}]	0.525	-10.6	0.642	9.6
D [m^2/s]	0.525	-10.5	0.642	9.5
a	0.630	7.4	0.543	-7.5

4. Conclusions

Sensitivity of the propagation velocity of the activation front was examined with respect to parameters of the modified FitzHugh-Nagumo atrial tissue model. The most pronounced effect on the propagation velocity was observed for parameters: c_1, that relates mainly to conductivity of sodium membrane channels and rate of their opening; k, that relates mainly to the speed of all reactions (depending e. g. on temperature); a, relating to the excitation threshold and D, determined mainly by the tissue conductivity. The effect of other examined parameters was less than 1 %. The differences in activation propagation velocities in SLAB 1 and SLAB 2 correspond to different shapes of activation fronts, wherein the plane activation front reached higher velocities than the convex activation front.

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