Detailed Uncertainty Analysis of the Tricept Kinematic Structure

[1]M. Omachelová, E. Kureková, [2]M. Halaj, [3]I. Martišovitš

[1] Faculty of Mechanical Engineering, Slovak University of Technology, 812 31 Bratislava, Slovak Republic

[2] Bratislava, Slovak Republic

[3] Microstep spol. s.r.o, Čavojského 1, 841 04 Bratislava, Slovak Republic

Email: milada.omachelova@stuba.sk

Abstract. *The article investigates the theoretical aspects of the positioning accuracy of parallel kinematic structures (PKS), especially the accuracy of the Tricept type PKS. Unlike serially configured structures utilizing translating and rotating movement, parallel kinematic structures consist of telescopic drives that are joined by a solid platform. The functional relationship between the actuators and the resulting position coordinates is rather complex, because of the configuration of the kinematic system. The article provides a framework to analyze the influence of geometrical imperfections in the system using the law of uncertainties propagation, in order to determine the accuracy of the end effector. The approach may aid the design process of parallel kinematic structures by providing information on the theoretically achievable effector positioning accuracy.*

Keywords: Parallel kinematic structures, Tricept, positioning accuracy, Coordinates uncertainty

1. Introduction

Parallel kinematic structures (PKS) represent a non-conventional way for arrangement of movement elements, comparing to the widely used serial kinematic structures. They employ parallel arranged movement elements (telescopic rods, arms) that have one end located at a base frame and the second end is connected to a movable platform. Tricept belongs among the most known PKS [1]. It is a fixed platform connected with a movable platform via three driving telescopic rods and a not-driven central rod (see Fig. 1). Central rod is connected to a movable platform by a solid linkage; while it can move axially against the fixed platform (rotation of the central rod is prevented). Effector is usually connected to a movable platform, carrying the tools or technological heads.

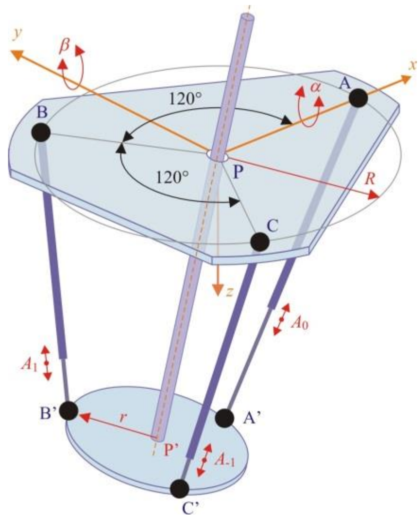

Fig. 1. Schematic representation of telescopic rods, joints and platforms

Servomotor located at the end of each telescopic rod enables extension of the rod by a ball screw and nuts. The skeleton together with a primary platform create a single kinematic element [1 to 3].

2. Influences that Affect Reaching of the Desired Position

Positioning accuracy of any manufacturing device represents the closeness between the actual reached position of the end effector and a programmed position, specified by the control system. For PKS, effector's endpoint is the point at the end of the central rod, respectively it is precisely defined point on the tool or technology head. In our case, the point P' is considered.

Based on theoretical analysis, one can summarize three types of errors affecting reaching of the desired position by PKS effector. The *geometrical errors* arise due to inaccuracies in manufacturing, inaccurate relative position of individual elements or due to wear of joints. The *stiffness errors* originate from elasticity of joints among individual elements as well as from flexures caused by own weights of individual elements or by an external load. Their magnitudes depend on the actual position of the effector. The *thermal errors* arise from thermal stress and dilatation of elements due to heat generated by internal or external sources, e.g. motors, bearings, etc. [2, 3].

3. Methodology for Determination of the Desired Position

If the device designer knows the theoretically achievable positioning accuracy, he has an important opportunity to influence critical pieces of equipment in the process of construction work. Uncertainties balance will help to identify the most significant influences on theoretically achievable positioning accuracy of the effector, which opens up the possibility of corrective interventions into the structure. Only geometrical influences on the overall uncertainty are considered in the paper.

Cartesian positions Q **=[**Q_x, Q_y, Q_z**]** of point Q_q (relative to "static" coordinate system bound with static platform (relative to point P), when (in general) angles α, β and shift z are nonzero) we obtain by applying transformations.

Matrix notation of transformation is q+ze3$Q = \boldsymbol{O}_y(\beta) \cdot \boldsymbol{O}_x(\alpha) \cdot (\boldsymbol{q} + z\boldsymbol{e}_3)$ that can be itemized as

$$\begin{pmatrix} Q_x \\ Q_y \\ Q_z \end{pmatrix} = \begin{pmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{pmatrix} . \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{pmatrix} . \begin{pmatrix} q_x \\ q_y \\ q_z + \mathrm{z} \end{pmatrix} \tag{1}$$

We can find their appropriate linear combinations to get the simplest relations equivalent to the relations of telescopic rods lengths. Three equations can be obtained in this way

$$\frac{-A_0^2 - A_1^2 - A_{-1}^2}{3} + r^2 + R^2 + z^2 - r.R.\cos\alpha - r.R.\cos\beta = 0 \tag{2}$$

$$-\frac{A_{-1}^2 - A_1^2}{\sqrt{3}} + 2R.z.\sin\alpha + r.R.\sin\alpha.\sin\beta = 0 \tag{3}$$

$$\frac{2A_0^2 - A_1^2 - A_{-1}^2}{3} + 2R.z.\cos\alpha.\sin\beta + r.R.\cos\beta - r.R.\cos\alpha = 0 \tag{4}$$

Let us denote the left sides of equations (2) to (4) as functions L_1, L_2, L_3 that depend on parameters $A_0, A_1, A_{-1}, \alpha, \beta, z, r, R$. We will consider the movement of the point Q in time t that will be limitedly close to 0 and parameters $A_0, A_1, A_{-1}, \alpha, \beta, z, r, R$ will depend on time t as well. If partial derivation of left sides of equations (2) to (4) following equation is obtained

$$\boldsymbol{W}_{3\times3} \quad \boldsymbol{M}_{3\times5}\boldsymbol{W}_{5\times1} + \boldsymbol{W}_{3\times5}\boldsymbol{W}_{5\times1} = \boldsymbol{0} \tag{5}$$

where

$$\boldsymbol{W}_{3\times3} = \begin{pmatrix} \frac{\partial L_1}{\partial\alpha} & \frac{\partial L_1}{\partial\beta} & \frac{\partial L_1}{\partial z} \\ \frac{\partial L_2}{\partial\alpha} & \frac{\partial L_2}{\partial\beta} & \frac{\partial L_2}{\partial z} \\ \frac{\partial L_3}{\partial\alpha} & \frac{\partial L_3}{\partial\beta} & \frac{\partial L_3}{\partial z} \end{pmatrix}; \ \boldsymbol{W}_{3\times5} = \begin{pmatrix} \frac{\partial L_1}{\partial A_0} & \frac{\partial L_1}{\partial A_1} & \frac{\partial L_1}{\partial A_{-1}} & \frac{\partial L_1}{\partial r} & \frac{\partial L_1}{\partial R} \\ \frac{\partial L_2}{\partial A_0} & \frac{\partial L_2}{\partial A_1} & \frac{\partial L_2}{\partial A_{-1}} & \frac{\partial L_2}{\partial r} & \frac{\partial L_2}{\partial R} \\ \frac{\partial L_3}{\partial A_0} & \frac{\partial L_3}{\partial A_1} & \frac{\partial L_3}{\partial A_{-1}} & \frac{\partial L_3}{\partial r} & \frac{\partial L_3}{\partial R} \end{pmatrix}; \ \boldsymbol{W}_{5\times1} = \begin{pmatrix} \dot{A}_0(0) \\ \dot{A}_1(0) \\ \dot{A}_{-1}(0) \\ \dot{r}(0) \\ \dot{R}(0) \end{pmatrix}.$$

Matrix $\boldsymbol{M}_{3\times 5}$ we express by the relation (5):

$$\boldsymbol{M}_{3\times 5} = -\boldsymbol{W}_{3\times 3}^{-1} \times \boldsymbol{W}_{3\times 5} \tag{6}$$

When multiplying the equation (1) from left by matrix $\boldsymbol{O}_y^T(\beta)$ and adjustment we get

$$-O_y^T(\beta).Q + O_x(\alpha).(q + ze_3) = 0 \tag{7}$$

Left sides (7) is matrix $\boldsymbol{H}$. Let

$$\boldsymbol{F}_{3\times 3} = \left(\frac{\partial H}{\partial Q_x}, \frac{\partial H}{\partial Q_y}, \frac{\partial H}{\partial Q_z} \right);\ \boldsymbol{G}_{3\times 3} = \left(\frac{\partial H}{\partial \alpha}, \frac{\partial H}{\partial \beta}; \frac{\partial H}{\partial z} \right), \text{ then } \boldsymbol{M}_{3\times 3} = \boldsymbol{F}_{3x3}^{-1}.(-\boldsymbol{G}_{3x3})$$

$$\boldsymbol{A}_{3\times 5} = \boldsymbol{F}_{3x3}^{-1}.(-\boldsymbol{G}_{3x3}).\boldsymbol{W}_{3x3}^{-1}.(-\boldsymbol{W}_{3x5}) = \boldsymbol{M}_{3x3}.\boldsymbol{M}_{3x5} \tag{8}$$

Matrix $\boldsymbol{A}_{3\times 5}$ from (8) is used for calculation of estimates of uncertainties of indirectly measured. Covariance matrix of those estimates is

$$\boldsymbol{U}_y = \boldsymbol{A}\boldsymbol{U}_x\boldsymbol{A}^{\mathrm{T}} \tag{9}$$

where matrix $\boldsymbol{U}_x$ is diagonal a known covariance matrix of the random vector $\boldsymbol{x} = (x_1, x_2, x_3, x_4, x_5) = (A_0, A_1, A_{-1}, r, R)$, where u_{x_i} is standard uncertainty of the estimate x_i of quantity X_i, $i = 1, 2, \dots 5$, $u_{x_{i,j}}$ is covariance among estimates x_i and x_j, $i = 1, 2, .., 5$, $j = 1, 2, .., 5$. Uncertainty of position of any point Q in the workspace can be calculated, if the matrix $\boldsymbol{U}_x$ is known.

Let $\boldsymbol{U}_x$ be a known constant symmetric matrix of 5×5 type, and $\boldsymbol{U}_y$ be an unknown symmetric matrix of 3×3 type that we want to determine and is given by (9). It is clear that the matrix $\boldsymbol{U}_y$ is correlated to the position of the point $Q[Q_x, Q_y, Q_z]$. We want to find the intervals for values of the matrix $\boldsymbol{U}_y$, when considering that Q_x, Q_y, Q_z may take any value, depending on how the reference point Q moves in some regular subspace (let it be a cube for purposes of this estimate, see Fig. 7) of the overall workspace.

If we fix the angles α and β, the virtual *beam* arises in cube, along which the reference point will move. It is sufficient to evaluate the expression for a particular beam only in the roots of this polynomial (if they overlap with workspace) and also in the endpoints of the beam, defined by the workspace borders. Among them we find the minimum and maximum, which will form the search interval for the selected element of matrix $\boldsymbol{U}_y$, for fixed angles α and β and a base matrix $\boldsymbol{U}_x$.

Impact of each element of the matrix $\boldsymbol{U}_x$ can be displayed using a three-dimensional function (see Fig. 2 until Fig. 6). Search estimate of the matrix $\boldsymbol{U}_y$ is obtained as a matrix of ordered pairs (minimum and maximum impacts of components of the matrix $\boldsymbol{U}_x$).

Using the software system MATHEMATICA, we created a program to search the entire workspace (or its subset thereof) and to estimate the matrix $\boldsymbol{U}_y$. To do so, the matrix $\boldsymbol{U}_x$ must be specified and the required division of the workspace must be selected.

For example, for matrix

$$\boldsymbol{U}_x = \left(\left(0{,}005/\sqrt{2}\right)^2, \left(0{,}005/\sqrt{2}\right)^2, \left(0{,}005/\sqrt{2}\right)^2, \left(0{,}01/\sqrt{2}\right)^2, \left(0{,}01/\sqrt{2}\right)^2 \right) \tag{10}$$

we found the estimate of $\boldsymbol{U}_y$

$$\begin{pmatrix} 0.0000185 & -7.0896\times 10^{-6} & -7.3262\times 10^{-6} \\ -7.0896\times 10^{-6} & 0.0000185 & -7.3394\times 10^{-6} \\ -7.3262\times 10^{-6} & -7.3394\times 10^{-6} & 0.0000038 \end{pmatrix} \le \boldsymbol{U}_y \le \begin{pmatrix} 0.0000368 & 7.0896\times 10^{-6} & 7.2188\times 10^{-6} \\ 7.0896\times 10^{-6} & 0.0000370 & 7.3394\times 10^{-6} \\ 7.2188\times 10^{-6} & 7.3394\times 10^{-6} & 0.0000138 \end{pmatrix} \tag{11}$$

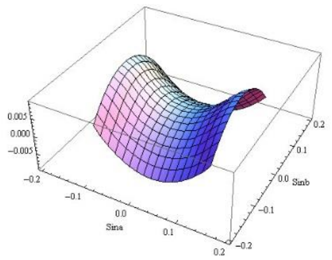

Fig. 2. Influence of element $\boldsymbol{U}_x[1,4]$ on maximum value of the matrix $\boldsymbol{U}_y[1,1]$

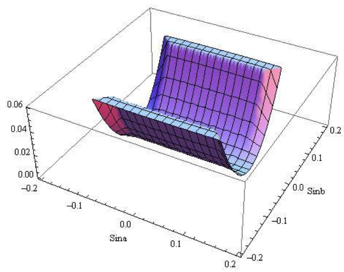

Fig. 3. Influence of element $\boldsymbol{U}x[5,5]$ on maximum value of the matrix $\boldsymbol{U}y[1,1]$

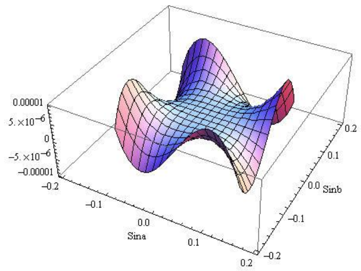

Fig. 4. Influence of element $\boldsymbol{U}x[4,4]$ on minimum value of the matrix $\boldsymbol{U}y[1,2]$

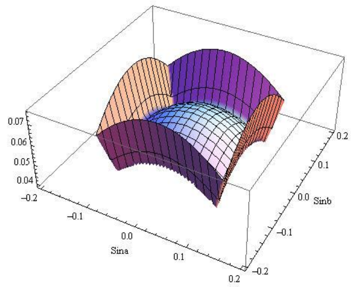

Fig. 5. Influence of element $\boldsymbol{U}x[5,5]$ on minimum value of the matrix $\boldsymbol{U}y[3,3]$

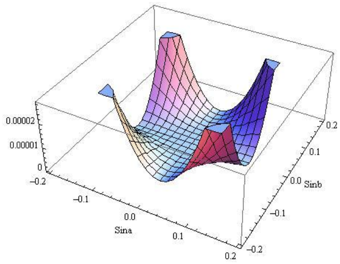

Fig. 6. Influence of element $\boldsymbol{U}x[4,4]$ on minimum value of the matrix $\boldsymbol{U}y[2,2]$

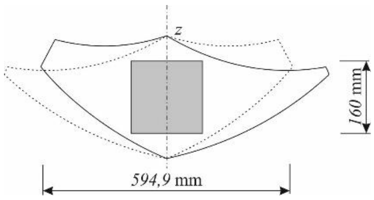

Fig. 7. Scheme of the cube that represents the biggest regular object in the workspace

4. Conclusions

This paper analyzed various issues related to the control of structures with parallel kinematics, especially that relating to the accuracy of positioning. The function describing the lengthening and shortening of the individual telescopic drives and the desired setpoint is non-linear. Because of this, the equations cannot be partially derived, making the uncertainty analysis unfeasible. In order to overcome this difficulty, the employment of an approach using infinite geometrical changes in the parameters is suggested. The limit variables for uncertainties were calculated here, suggesting that the achievable positioning accuracy is not constant for all setpoints within the workspace of the Tricept device.

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