The Envisaged New SI: Challenges for Precision Engineering

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Abstract. *In 1983 the definition of the SI unit of length, the metre, was defined by fixing the numerical value of a natural constant, namely the speed of light in vacuum c_0, without any uncertainty. The numerical value was derived from the most precise measurement values for c_0, known at the time of the definition. For the envisaged New SI, a similar approach is proposed for the re-definition of the SI units of mass, amount of substance, temperature and electrical current. For the definition of these SI units, the numerical values of the following natural constants are proposed to be fixed: Planck constant h, Avogadro constant N_A, Boltzmann constant k and elementary charge e. In this contribution it will be described how progress in precision engineering is needed to further reduce the uncertainties of experiments for the determination of h, N_A and k as a necessary prerequisite for the New SI to become effective in 2018 as proposed by the General Conference for Weights and Measures.*

Keywords: New SI, Defining constants, Precision engineering, XRCD method, Watt balance

1. Introduction

The system of Weights and Measures was recognized as a necessary basis for science, agriculture, construction and trade from the very beginning of ancient civilization. The importance of a reliable system of Weights and Measures or generally speaking of a reliable System of Units is documented by the fact that it has always been regarded as a governmental task of high priority until today. Throughout history the definitions of the units for Weights and Measures usually also differed from state to state until at the end of the 18^{th} century the idea was followed in France to base the definition of the unit of length on a stable natural reference, which was chosen to be the circumference of the earth. The metre standard (*mètre des Archives*) was thus defined as one ten-millionth of the distance from the North Pole to the Equator [1]. Finally, in 1875 the metre convention was signed by 17 national states and at the first General Conference on Weights and Measures in 1889 copies of the international protoypes of the kilogram and the metre made from Pt-Ir were distributed among the member states of the metre convention. All length measurements had to be referred to the international metre prototype (Pt-Ir bar with x-shaped cross-section and engraved lines). Starting with Michelson already in 1889 the development and application of interferometry allowed to decrease the uncertainty of length measurements. This was the reason that in 1960 the prototype-based definition was replaced by a wavelength-based definition of the metre using the orange spectral line of the ^{86}Kr spectral lamp.

This development shows that the definition of units should be flexible enough to make benefit of technological developments and to circumvent possible limitations of material standards or material effects. The envisaged New SI will be a big step in this direction. In particular, it will allow to replace the remaining prototype-based definition of the unit of mass, the kilogram, by a definition which fixes the numerical value of a natural constant, namely the Planck constant.

2. International System of Units (SI)

In 1960 the 11th General Conference on Weights and Measures (CGPM) adopted the name *Système International d'Unités* (International System of Units, international abbreviation SI), for the recommended practical system of units of measurement. The SI base units are a choice of seven well-defined units which by convention are regarded as dimensionally independent: the metre, the kilogram, the second, the ampere, the kelvin, the mole, and the candela. Derived units are those formed by combining base units according to the algebraic relations linking the corresponding quantities [2]. Taking into account the historical development of the system of units, also the SI was thought not to be static but to evolve to match the world's increasingly demanding requirements for precise measurements.

One important change to the SI was made in 1983 when the unit of length, the metre was defined by fixing the - at this time - best known numerical value of a natural constant, the speed of light in vacuum c_0, namely, 299 792 458 metre per second exactly, i.e. without attributing an uncertainty for c_0.

Envisaged "New SI"
This concept of defining an SI base unit by fixing the numerical value of a properly chosen natural constant or at least a so-called defining constant also inspired the envisaged "New SI" or the possible future revision of the SI.

In the "New SI" four of the SI base units, namely the kilogram, the ampere, the kelvin and the mole, will be redefined in terms of invariants of nature; the new definitions are proposed to be based on fixed numerical values of the Planck constant (h), the elementary charge (e), the Boltzmann constant (k), and the Avogadro constant (N_A), respectively, see figure 1.

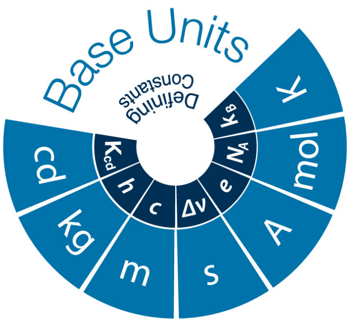

Fig. 1. Scheme of the base units and the defining constants in the New SI.

Different highly sophisticated experiments have been performed to realize precise measurements of the above mentioned natural constants. However, "while remarkable progress has been made over the last few years, the conditions for adopting the redefinitions, as set by the CGPM at its 23rd meeting (2007), have not yet been fully met" [3].

It is important to note, that *progress in precision manufacturing and precision engineering* is necessary to enable the envisaged New SI. In this contribution some examples of precision machining tasks and achievements in precision dimensional metrology will be discussed.

The most demanding conditions are set for the determination of the Planck constant h, which in the New SI will be used to define the unit of mass, the kg. The Consultative Committee for Mass (CCM) has formulated different prerequisites [4] before the new definition of the unit of mass could be put into place as well as a roadmap of different necessary steps to be achieved to reach the target until 2018. The updated CCM roadmap activity has been endorsed by the the CIPM at its 102nd meeting in 2013 and the CGPM at its 25th meeting in 2014 [5].

One of the conditions defined by the CCM is that at least three independent experiments, including work from watt balance and XRCD experiments, yield consistent values of the

Planck constant with relative standard uncertainties not larger than $5 \cdot 10^{-8}$ and that at least one of these results should have a relative standard uncertainty not larger than $2 \cdot 10^{-8}$.

3. Precision experiments for *h*

To determine the value of the Planck constant with the above mentioned uncertainties two different approaches are followed in metrology institutes worldwide, namely the watt balance experiments and the x-ray crystal density (XRCD) experiments [6].

In the watt balance experiments the gravitational force of a mass standard is compared with compensating electromagnetic forces generated by a coil in a static homogenous magnetic field in two different configurations, namely with a stationary coil carrying a defined current (weighing phase) and with the same coil moving at constant speed, thus inducing a constant voltage over the coil (moving phase), see figure 2 [7].

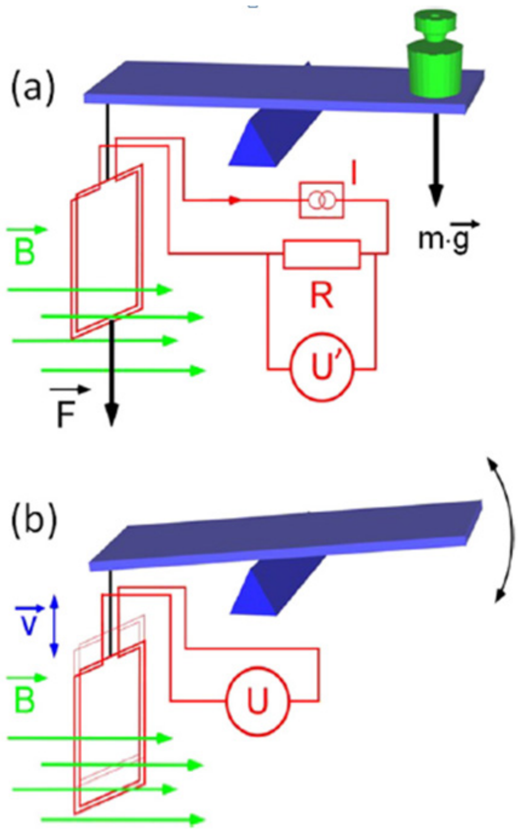

Fig. 2. a) static mode: the electromagnetic force acting on the current-carrying coil is balanced against the weight of the test mass m; b) dynamic mode: the coil is moved in the vertical direction through the magnetic field and the induced voltage U is measured. [7]

The precise alignment of the components of the watt balance as well as the control and interferometric measurement of the linear movement of the coil in the dynamic phase of the experiment are very demanding precision engineering challenges related to the watt balance experiments. In [8] an analysis on the alignment requirements was given for the watt balance operated at NRC. Different influences were analyzed, such as alignment of the laser beam, residual torques about horizontal axes and horizontal forces, alignment of the mass pan of the balance, determination of the Abbe offset, measurement and adjustment of the horizontal velocities, measurement of the angular velocities and changes in the coil position between phases. The resulting combined relative alignment uncertainty was estimated to be $5.5 \cdot 10^{-9}$. Recently, a new measurement value for the Planck constant *h* using the NRC watt balance has been reported with a relative uncertainty of $1.8 \cdot 10^{-8}$ [9], which would satisfy the condition set by CCM with respect to the smallest uncertainty of one from at least three independent experiments to be not larger than $2 \cdot 10^{-8}$.

In the XRCD method one uses a high purity, highly enriched ^{28}Si sphere (diameter of 93 mm and mass of about 1 kg) to precisely determine the Avogadro constant (N_A). The Avogadro constant links the atomic to the macroscopic world and is determined by the ratio of the density of an atomic unit cell of the ^{28}Si sphere and the density of the macroscopic sphere.

The measurement of the Avogadro constant is based on $N_A = n \cdot M / (\rho \cdot a^3)$, where $n = 8$ is the number of atoms per unit cell of a silicon crystal and ρ, M and a are the density, molar mass and lattice parameter, respectively. Precise measurements of the molar mass, the Si lattice parameter as well as the mass and the volume of the Si sphere are thus required to determine N_A. Because the molar Planck constant $h \cdot N_A$ is known from γ-spectroscopy experiments with an uncertainty of $7 \cdot 10^{-10}$ [10], a precise measurement of N_A also provides a precise value of h. A value for N_A using the XRCD method with a standard uncertainty of $3 \cdot 10^{-8}$ was published by the International Avogadro Consortium (IAC) in 2011 [11]. A new result of the IAC for N_A was recently measured with an improved standard uncertainty of $2 \cdot 10^{-8}$ [12].

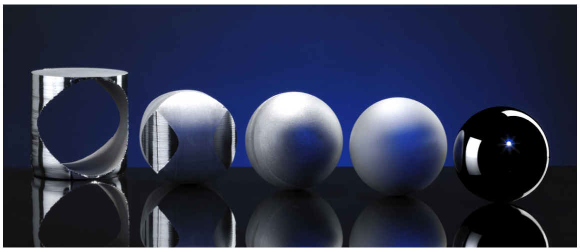

Fig. 3. Overview of different sphere manufacturing steps (hollowed ingot, cut form, turned, coarsely lapped and polished sphere).

In addition to the production requirements for the pure isotope enriched silicon single crystal material, also the manufacturing specifications for surface quality (average roughness values below 0.3 nm) and spherical form (roundness deviation below 30 nm in amplitude) of the macroscopic silicon spheres are very demanding, see figure 3 [13]. The biggest uncertainty contribution - about 2/3 of the total budget - so far is due to the volume determination of the silicon spheres. A special spherical Fizeau interferometer has been developed for this purpose, see figure 4 for a schematic drawing [14] as well as techniques to characterize the influence of different surface layers on the determination of N_A [15].

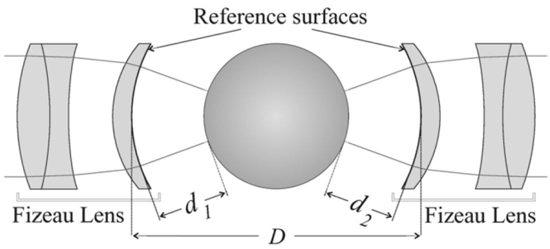

Fig. 4. Schematic drawing of the central part of the interferometer. The measured quantities are the distances between the sphere surface and the reference surfaces, d_1 and d_2, and the length of the empty etalon D.

Figure 5 shows a graphical representation of the resulting radius topography from all measurements performed on one of the silicon spheres manufactured along the process chain referred to in figure 2. One can clearly recognize the high symmetry with the characteristic of a rhombic dodecahedron – in the magnitude of several nanometers. This typical appearance of a cubic crystal is worked out in a unique and clear form (see figures 5, 6) and could also be repeated on other spheres.

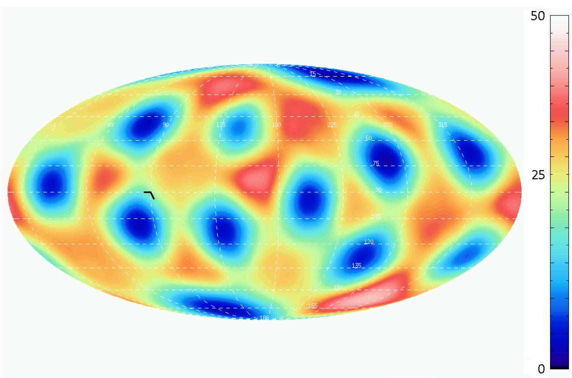

Fig. 5. Radius topography of silicon sphere PTB_11-01.

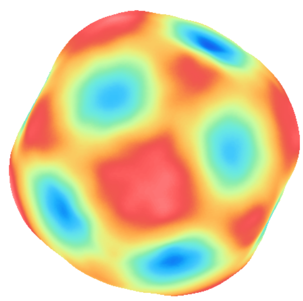

Fig. 6. Deviations from roundness of sphere PTB_11-01, 10^6-fold enhanced.

Although today three independent experiments for *h* have been published with uncertainties below $5{\cdot}10^{-8}$ and one experiment with an uncertainty below $2{\cdot}10^{-8}$ which also seem to be fairly consistent, one should not forget, that the CCM also requires procedures for the future realization and dissemination of the kilogram, as described in the *mise en pratique*, have been validated in accordance with the principles of the CIPM – MRA. This validation process of the procedures will need additional time as foreseen in the CCM roadmap.

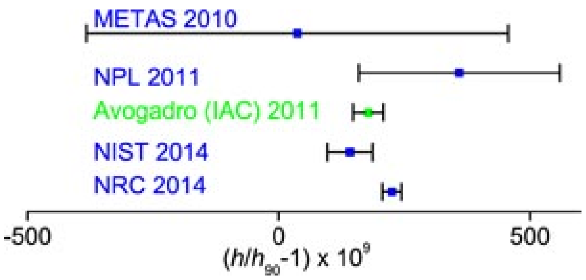

Fig. 7. Measurement results for Planck's constant *h*.

Figure 7 shows results and standard uncertainties of different experiments (watt balances and Avogadro (IAC)) for determination of *h*, as published in [16].

For an updated analysis of the measurement results on the NIST-3 watt balance over the last decade see [17].

4. Precision experiments for *k*

For precise determination of the Boltzmann constant *k*, different approaches are followed at NMI's worldwide. The most promising method with smallest achievable relative standard uncertainties below 10^{-6} is called acoustic gas thermometry (AGT). One experiment uses acoustic waves in a noble gas inside a precisely manufactured slightly ellipsoidal resonator to determine *k* [17], see figure 8. The small uncertainty obtained with this experiment was reported to be $u_R = 0.71 \cdot 10^{-6}$. In addition to other AGT experiments using spherical or slightly ellipsoidal resonators, there is another experiment which uses precisely manufactured cylindrical resonators for the AGT experiment [18].

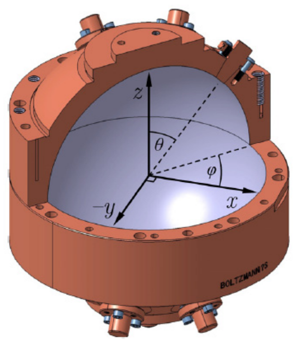

Fig. 8. Manufactured resonator for the NPL Boltzmann experiment [17].

One of the other independently developed methods for determination of *k* is based on dielectric-constant gas thermometry DCGT [19]. Here, the dielectric constant of a noble gas is measured under different pressures using a precise pressure balance. The calibration of the pressure balance is traceable to dimensional characterizations (diameter and form) of the effective area of cylindrical piston-cylinder pressure gauges, as shown in figure 9. Standard uncertainties of three-dimensional data of 8 nm for pistons and 16 nm for cylinders were obtained by a combination of high precision form and diameter measurements [21].

Fig. 9. Piston cylinder pressure gauge used for the determination of the Boltzmann constant *k* by means of the DCGT method.

5. Discussion

Some of the contributions from precision manufacturing and precision engineering for the progress of experiments in fundamental metrology aiming at the envisaged New SI were addressed. It has been shown how progress in manufacturing chains as well as in alignment and characterization of critical components of experiments contribute to the required further reduction of uncertainty of different experiments for determination of more precise numerical values of natural constants needed for the proposed definitions of the New SI.

Although today the formal requirements of the CCM with respect to the uncertainty and consistency of at least three independent experiments for determination of the Planck's constant *h* seem to be fulfilled, it will be of importance to follow the results of other watt balance experiments as well as repeated Avogadro experiments with a second lot of ^{28}Si material, which are under preparation. It is also of importance to maintain the established manufacturing chains and the described precision experiments for future use.

Acknowledgements

The author gratefully acknowledges the support of colleagues from PTB as well as external partners involved in the activities for the redefinition of the SI.

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