Soft Magnetic Objects in Homogeneous Magnetic Field of an NMR Imager, Mathematical Modelling and Experimental Evaluation

I. Frollo, A. Krafčík, P. Andris, J. Přibil, T. Dermek

Institute of Measurement Science, Slovak Academy of Sciences, Bratislava, Slovakia
Email: frollo@savba.sk

Abstract. *Soft magnetic samples with magnetic particles ingredients were placed into the homogenous magnetic field of an imager based on nuclear magnetic resonance. Theoretical computations based on a magnetostatic models were performed. For experimental verification an MRI 0.2 Tesla ESAOTE Opera imager was used. For modelling and experiments two samples - rectangular and circular were tested. The resultant images correspond to the magnetic field variations in the vicinity of the samples.*

Keywords: Soft Magnetic Objects, Mathematical Modelling, Magnetic Resonance Imaging, Magnetic Liquids, Magnetic Susceptibility

1. Introduction

Theoretically and experimentally was proved that every physical or biological object, which is inserted into magnetic field, deforms this field. Objects attract or detract the magnetic line of force, depending on the object substance: ferromagnetic, paramagnetic or diamagnetic.
The specific quantity is the magnetic susceptibility, which is defined as a change of magnetization in dependence on magnetic field intensity. Values of susceptibility defined as $\chi_i = (dM / dH)_0$ i.e. the slope at the origin, (M - magnetization, H - magnetic field, whereas $M = \chi H$) range from around -10^{-5} in very weak magnetic materials up to values of around $+10^6$ in ultra-soft ferromagnets [1].

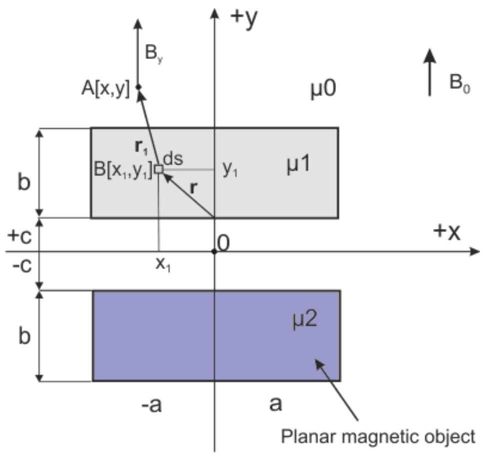
Imaging methods based on magnetic resonance principles are capable to detect, to measure and to image these deformations [2], [3]. For the purpose of mathematical models based on integral equations, vector potentials are applied. The calculation results are in a form of analytical expressions that for particular evaluation need perfect computational system and considerable computing time [4].
In this paper we try to describe the magnetic field distribution by mathematical modelling with an orientation to the simplest rectangular and circular objects.

2. Subject and Methods

Let us assume that a ferromagnetic or paramagnetic object is placed into the homogeneous magnetic field of an MRI imager. The homogeneous magnetic field near the sample is deformed. For simplicity and easy experimental verification a double rectangular and circular objects were selected and theoretically analyzed.

A. Rectangular Samples

For the purpose of our simple example we suppose that the soft magnetic planar layer is positioned in the x-y plane of the rectangular coordinate system and the thickness of the layer is neglected. According to Fig.1, we suppose the layers are limited by dimensions of 2a and b, with the left-right symmetry. The layers are moved in y direction by distances +c and –c,



ds[x1,y2] is an elementary surface element. The basic magnetic field B_0 of the MR imager is parallel with the y-axis. The task is to calculate the $B_y(x,y)$ component of the magnetic field in the point A[x,y].

Fig.1. Fundamental configuration of the two magnetic planar rectangular layers. Samples are positioned in x-y plane of the rectangular coordinate system. The thickness of the layers is neglected.

In planar approximation static magnetic objects (linear and isotropic) without any driving current can be described by Laplace equation: $\nabla^2 A(x,y,z) = 0$, (1)

for nonzero z-component of magnetic vector potential $\mathbf{A} = [0,0,A]$, and by boundary conditions at each interface between two media: $\mathbf{n}_{\mathrm{II}} \cdot (\mathbf{B}_{\mathrm{I}} - \mathbf{B}_{\mathrm{II}}) = 0$, $\mathbf{n}_{\mathrm{II}} \times (\mathbf{H}_{\mathrm{I}} - \mathbf{H}_{\mathrm{II}}) = \mathbf{0}$, (2) (material domains I and II, and $\mathbf{n}_{II}$ is an outward normal from domain II) and outer boundary conditions.

Laplace equation is solvable analytically only for the simplest problems. For rectangular sample, these equations were therefore solved numerically by finite element method (FEM). In this way it was calculated distribution of magnetic flux density field of two parallel bars with rectangular cross-section in x-y plane, with constant permeability μ_k (i.e. for *k*-th bar material domain) in originally homogeneous magnetic field $\mathbf{B}_0 = [0, B_0, 0]$ perpendicular to longitudinal axis of bars, see Fig.2, left.

For experimental evaluation of the mathematical modelling, we have chosen a simple laboratory arrangement by application of the magnetic resonance imaging methods. For sample positioning, a rectangular plastic vessel or holder with tap water was used. Two samples - rectangular vessels filled with doped water (several drops of magnetic liquid based on Dextran) were placed to the centre of the holder. Dimensions of the samples: 50x10 mm, distance between samples: 5 mm. Thickness of the walls of sample vessels: 1 mm. Resultant image is in Fig.2, right.

The contrast of the imaged samples corresponds to the real values of μ1, μ2, and μ0.
Walls of sample vessels with liquid substances are imaged with black colour, no MRI signal.

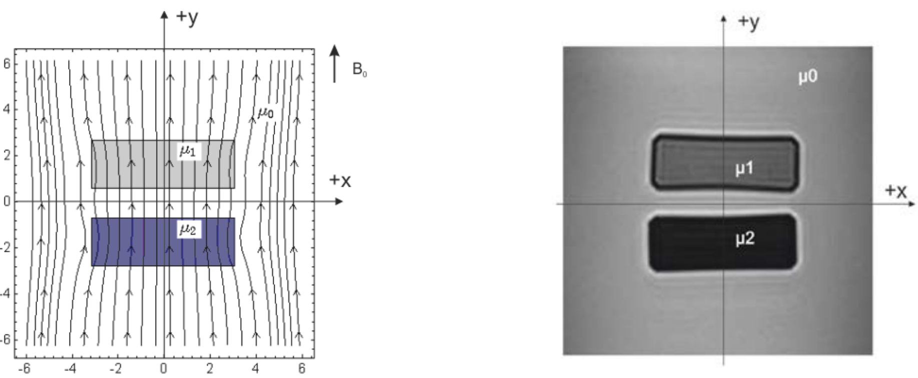


Fig. 2. Left: Calculated magnetic field distribution of the homogeneous magnetic field B_0, lines of force, affected by 2 parallel magnetic bars with relative permeabilities: μ1=2 , μ2=3 and environment μ0. Right: NMR image of two samples (rectangular vessels filled with doped water in the holder with tap water) using GRE imaging sequence, TR = 440 ms, TE = 10 ms. Thickness of the imaged layer: 2 mm.

B. Circular Samples

For circular samples, 3 cylinder vessels placed in a rectangular plastic holder filled with the tap water were used. Cylinder vessels were filled with distilled water doped by several drops of magnetic liquid based on Dextran (polysaccharide).
According to Fig.3, we suppose three circular concentric vessels. Relative permeabilities: μ1, μ2, environment permeability: μ0.

From Fig.3 the position vector can be expressed as: $r = \sqrt{x^2 + y^2}$. (3)

For vector potential we can write [5]: $A_k = (C_k\, r + \frac{D_k}{r})\, \frac{x}{r}$, (4)

where C_k and D_k are constants calculated for every region for relative permeabilities corresponding to the concrete experiment.

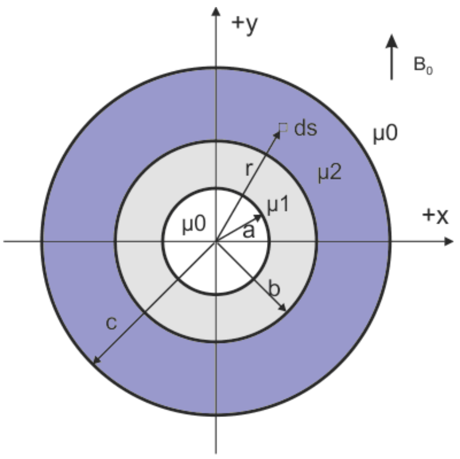


Fig. 3. Basic configuration of the magnetic planar layer circular sample positioned in x-y plane of the rectangular coordinate system.

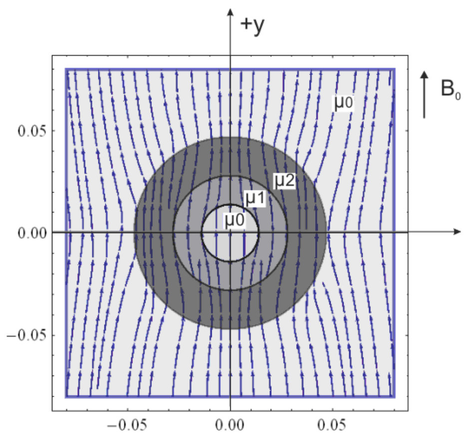
Resultant field will be: $H_r = \mathrm{Curl}\,[A_r, \{x, y\}]$. (5)

Using function:

StreamPlot[{H_r}, {x,-0.08, 0.08}, {y,- 0.08, 0.08}]

it is possible to draw the lines of force of the examined objects, relative values, Fig.4, left.

For experimental evaluation of the mathematical modelling a rectangular plastic vessel - holder with tap water inside was chosen. Two samples - circular vessels filled with doped water were placed to the centre of the holder. The central vessel was filled with tap water. Diameters of the vessels: 18, 40, 56 mm, wall thickness: 1 mm. Resultant NMR image is depicted in Fig.4, right.



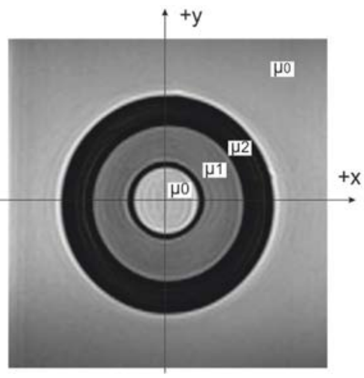


Fig. 4. Left. Mathematical model of the distribution of homogeneous magnetic field H_0, lines of force affected by 2 concentric cylinders with relative permeabilities: μ1=2 and μ2=3, μ0 is an environment permeability.
Right. NMR image of two samples (circular vessels filled with doped water in the rectangular vessels - holder filled with tap water) using GRE imaging sequence: TR = 440 ms, TE = 10 ms. Thickness of the imaged layer: 2 mm.

For experimental verification an MRI 0.2 Tesla ESAOTE Opera imager (Esaote, Genoa, Italy) with vertical orientation of the basic magnetic field was used.

3. Results and Discussion

The goal of this study was to mathematically describe and experimentally depict simple rectangular and circular objects - vessels filled with doped water by very diluted magnetic liquids. Imaging method based on MR principles was used for interpretation of analyzed samples.

Mathematical analysis of rectangular and circular objects, representing a shaped magnetic soft layer, showed theoretical possibilities to calculate magnetic field around any type of sample. Our mathematical model proved that it is possible to map the magnetic field variations - line of force - and to image the specific structures of selected samples placed into a special plastic holders. The calculations were performed with relative values of input quantities, permeabilities and dimensions. The mathematical model was described in the form of general formulas. To present the detailed analytical form of resultant equations would exceed the range of this paper.

For experimental presentation a classical gradient echo measuring sequences were used. The resultant MR images are encircled by narrow stripes that optically extend the width of the sample. This phenomenon is typical for susceptibility imaging, when one needs to measure local magnetic field variations representing sample properties [3].

4. Conclusions

Our experimental results are in good correlation with the mathematical simulations in spite of using relative quantities. This validates the possible suitability of the proposed method for detection of weak magnetic materials using the MRI methods. Presented images of thin objects indicate perspective possibilities of this methodology even in the low-field MRI.

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